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The Efficient Design of Adhesive Bonded Joints

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A concise method of analysis is used to study the numerous parameters influencing the stress distribution within the adhesive of a single lap joint. The formulation includes transverse shear and normal strain deformations. Both isotropic or anisotropic material systems of similar or dissimilar adherends are analysed. Results indicate that the primary Young's modulus of the adherend, the overlap length, and the adhesive's material properties are the parameters most influential in optimizing the design of a single lap joint.

INTRODUCTION

The objective of this paper is to explicitly identify those parameters which have a significant influence on the stress distribution in an adhesive bonded joint and in the adjoining adherends. Understanding the influence of these various parameters, one will then be able to design an optimum joint for specific design requirements.

R. E. Watson¹ writes that, "The next two decades will surely see dramatic advances in structures as compared to those experienced over the last 30 years. Improved titanium alloys and advanced high strength composites, with more strength per pound than aluminum, will be the principal materials used". Furthermore, he states, "New bonding techniques will gradually replace riveting in many applications, permitting greater design stresses and more efficient distribution of the materials".

With increased use of filamentary composite materials, methods of analysis for structures of homogeneous isotropic materials must be modified drastically. Not only must new methods be developed to account for anisotropic materials, but these methods must account for laminated construction. Further, because of the larger ratio of the in-plane elastic modulus to the

transverse shear modulus, unlike contemporary methods, these new analyses must include the effects of transverse shear and transverse normal strain deformation.

In Ref. (2), a comprehensive method of analysis has been developed for the linear elastic response of two laminated plates joined through a bonded single lap joint, subjected to in-plane loads. The effects of the anisotropic properties of each lamina, the lamina fiber orientation, adhesive thickness, thermal effects, and lap length are each included. Transverse shear deformation and transverse normal strain effects are, of course, included in the formulation. In addition, the mechanical properties of the adhesive material as they exist in the thin lap joint configuration are also included.

The analysis enables one to determine accurately the state of stress within both the joint adhesive and the adherends for similar and dissimilar adherends of either isotropic or anisotropic materials.

A laminated plate element (Figure 1) is the building block for developing the method of analysis. The plate element used provides for moment, shear and axial loads, plus normal distributed loads and surface shear stresses. To solve the single lap joint problem, requires satisfying twenty-six boundary conditions.

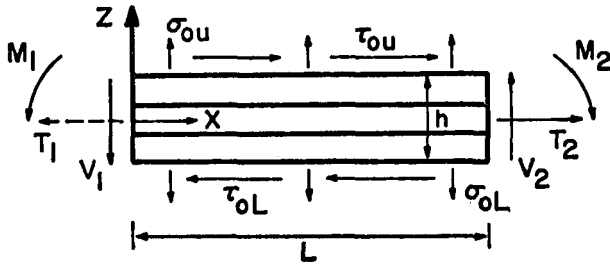


FIGURE 1 Laminated plate element.

With the inclusion of transverse shear deformation and normal strain effects, an accurate shear stress distribution in the adhesive is obtained; namely, the shear stress goes to zero at the edge of the lap, reaches a maximum value a short distance away from the edge and diminishes somewhat further into the joint interior. Transverse normal stresses in the adhesive do reach a maximum at the edge of the lap.

METHOD OF ANALYSIS

The assumptions and/or limitations of the method of analysis developed in Ref. (2) are as follows:

- 1) The laminated adherends are symmetric about their own midsurface (i.e. no bending-stretching coupling in the adherend).

2) Plane strain exists in the adherend, in a direction perpendicular to the load (i.e. in the x - z plane).

3) Each ply or lamina in each adherend is orthotropic.

4) The elastic mechanical properties of the adhesive are accounted for.

5) The transverse shear stress distribution in each lamina is assumed to be parabolic across its thickness.

6) Both shear and transverse normal stresses are accounted for in the adhesive, vary in the load direction, but do not vary in the thickness direction.

7) The adhesive thickness is much smaller than the adherend thickness.

8) Transverse shear deformation and transverse normal strains are accounted for in each adherend.

9) Thermal strains are accounted for.

The constitutive relation for the K th lamina is:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix}_K = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} & 0 & 0 & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{23} & 0 & 0 & \bar{Q}_{26} \\ \bar{Q}_{31} & \bar{Q}_{32} & \bar{Q}_{33} & 0 & 0 & \bar{Q}_{36} \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} & 0 \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} & 0 \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{63} & 0 & 0 & \bar{Q}_{66} \end{bmatrix}_K \begin{bmatrix} \varepsilon_x = \varepsilon_{x_T} - \alpha_x \Delta T \\ \varepsilon_y = \varepsilon_{y_T} - \alpha_y \Delta T \\ \varepsilon_z = \varepsilon_{z_T} - \alpha_z \Delta T \\ \varepsilon_{yz} = \varepsilon_{yz_T} \\ \varepsilon_{xz} = \varepsilon_{xz_T} \\ \varepsilon_{xy} = \varepsilon_{xy_T} - \alpha_{xy} \Delta T \end{bmatrix}_K \quad (1)$$

Assuming a state of plane strain exists in the x - z plane, the equilibrium, strain-displacement and transverse shear stress relations are:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0; \quad \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} = 0. \quad (2, 3)$$

$$\varepsilon_x = \frac{\partial u}{\partial x}; \quad \varepsilon_y = \frac{\partial v}{\partial y} = 0. \quad (4, 5)$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \quad (\neq 0 \text{ as in classical plate theory}) \quad (6)$$

$$\varepsilon_{xz} = \frac{1}{2} \left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right] \quad (7)$$

$$\varepsilon_{yz} = \frac{1}{2} \left[\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right] = 0; \quad \varepsilon_{xy} = \frac{1}{2} \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right] = 0 \quad (8, 9)$$

$$\tau_{xz}|_K = \tau_{0L} F_1(z) + \tau_{0u} F_2(z) + [Q_{55}|_K F_3(z) + b_{55}|_K] \phi_x \quad (10)$$

where ϕ_x is an arbitrary function to be determined; b_{55} is a constant which guarantees the continuity of shear stress at the lamina interface and $F_i(z)$ are functions that describe the shear stress distribution throughout the adherend.

Employing laminated plate theory,³ the resultant moment, axial load and

transverse shear relations for an orthotropic laminate accounting for transverse shear and normal strain effects can be written. Having these fundamental relationships, one may now solve the single lap joint problem (Figure (2)) for similar or dissimilar adherends composed of isotropic or anisotropic material systems.

The governing equation for the adhesive shear stress (τ_0) is of the form:

$$\bar{C}_9 \frac{d^8 \tau_0}{dx^8} + \bar{C}_7 \frac{d^6 \tau_0}{dx^6} + \bar{C}_5 \frac{d^4 \tau_0}{dx^4} + \bar{C}_3 \frac{d^2 \tau_0}{dx^2} + \bar{C}_1 \tau_0 = g(x) \quad (11)$$

where $g(x)$ is a function of the temperature distribution in the x -direction.

The complete solution of this equation is:

$$\tau_0(x) = S_i e^{\lambda_i x} + \left(\begin{array}{c} \text{Particular} \\ \text{solution} \end{array} \right) \quad i = 1 \rightarrow 8 \quad (12)$$

The normal stress is given by:

$$\sigma_0(x) = -\beta_i \lambda_i S_i e^{\lambda_i x} + \frac{d}{dx} \left(\begin{array}{c} \text{Particular} \\ \text{solution} \end{array} \right) \quad i = 1 \rightarrow 8 \quad (13)$$

where S_i are the 8 independent boundary constants. λ_i are the roots of Eq. (11) and the particular solution is a function of the temperature distribution function. \bar{C}_i and β_i are material property parameters.

PARAMETRIC RESULTS

The type of failure one observes in a bonded joint is dependent on whether the joint is experiencing a static or fatigue loading condition. For example, while a given joint may fail in the adhesive due to peak shear and normal stresses at ultimate load, it very possibly could fail in fatigue in the adherend due to a high moment concentration factor at the edge of the joint induced by the joint eccentricity. Such a failure would depend on the materials being used, the mean fatigue load and the fatigue stress ratio which the adherend is experiencing.

Hypothetically, an adhesive bonded joint may fail due to static or fatigue loads in three distinct modes. The adhesive may fail due to high shear and normal stresses. The adherends may fail due to an axial load coupled with too large a moment at the joint edge or if the adherends are laminated, a ply in the adherend near the joint can fail by resin deterioration due to high interlaminar stresses (ref. 2). Thus, this study will explain why specific parameters significantly influence the type of failure a bonded joint experiences.

The baseline geometry and pertinent material parameters are given in Figure (2). The parameters from which all variations were made in this

study are: $T = 275$ lb, $L_1/H_{21} = 67$, $L_4/H_{31} = 42$, and overlap length = 0.61 inches. The effective shear and tensile moduli of the adhesive are 1500 psi and 9900 psi respectively. The ratio of the pertinent adherend material properties are $Q_{11(2)}/Q_{11(1)} = 1.7$, $Q_{55(2)}/Q_{55(1)} = 7.7$, $A_{11(1)}/A_{11(2)} = 0.372$ and $D_{11(1)}/D_{11(2)} = 0.147$. The adhesive thickness (η) = 0.003 inches. These data are typical of numerous adherend material combinations.

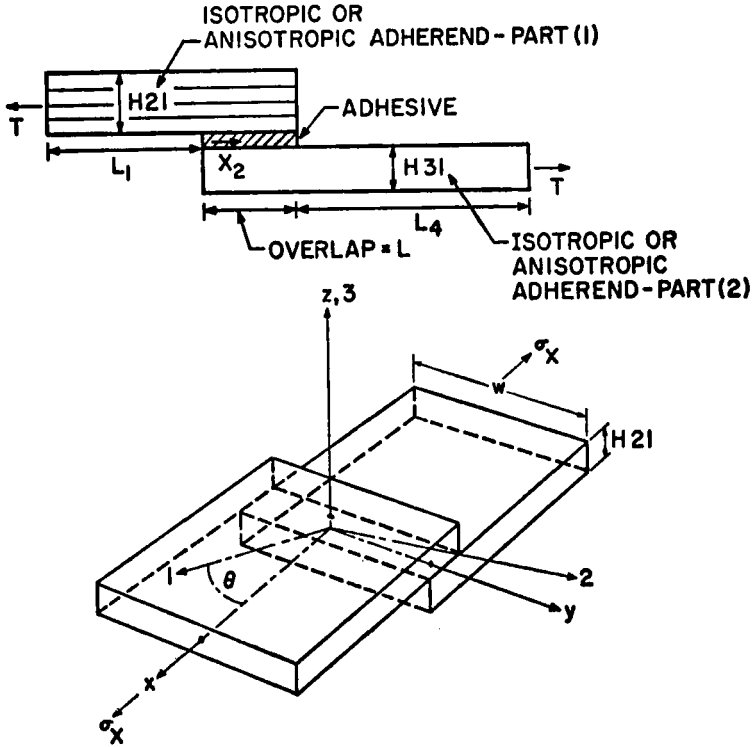


FIGURE 2 Baseline geometry.

Figure (3) shows the influence of varying the ratio of the primary elastic moduli (Q_{11}) of the two adherends. The ratio of $Q_{11(1)}$ to the effective shear modulus is held fixed and is 4.1×10^3 . Generally, the larger the disparity between the primary moduli, the greater the peak shear stress. Moreover, the larger the numerical values of the primary moduli, the more uniform the shear stress distribution in the adhesive becomes. The peak adhesive shear stress for identical adherends is the same at each edge with the distribution being symmetric about the centerline. For dissimilar adherends, the peak adhesive shear stress occurs just inboard of the edge where the adhesive joins the adherend of lesser in-plane stiffness.

* () The number in parentheses refers to the part in question in Figure (2).

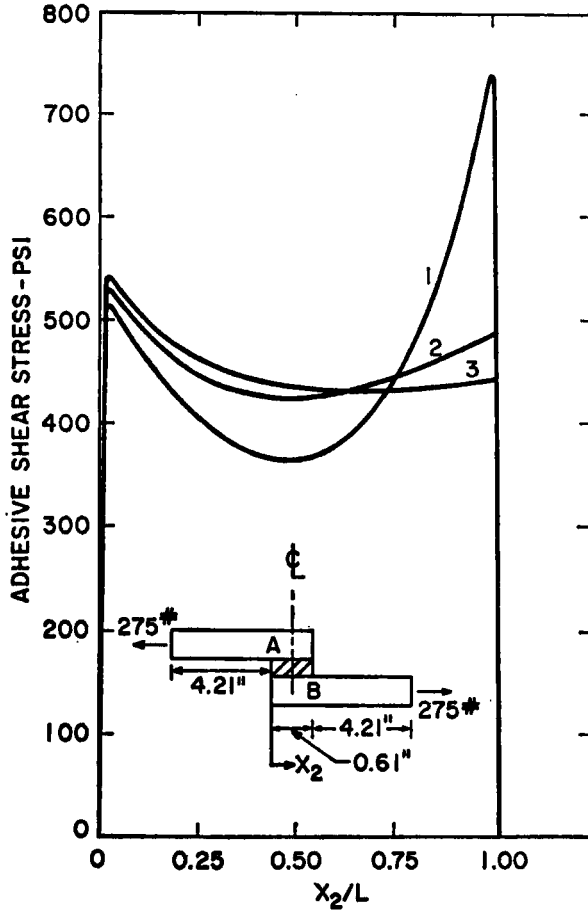


FIGURE 3 Curves 1, 2 and 3. The influence of the ratio of the primary moduli, $Q_{11(2)}/Q_{11(1)}$, of 0.148, 1.7, 4.42 respectively.

Contrary to this, the bending moment in the adherends, due to the joint eccentricity, occurs in the adherend of maximum flexural stiffness adjacent to the adhesive (point B). The difference in magnitude of the maximum moment in each adherend varies directly as the ratio of the flexural stiffness (i.e. D_{11}) in each adherend.

The reason the peak adhesive shear stress always occurs at the end adjacent to the least stiff adherend is that less stiffness results in greater differential deformation ($U_2 - U_1$) in the direction of the applied load. This deformation is related to the shear stress by:

$$\tau_0 = \frac{G_{\text{eff.}}}{\eta} (U_2 - U_1) \quad (14)$$

Figure (4) implies that as the ratio of the primary moduli of the adherends increases in magnitude, the peak value of normal stress in the adhesive decreases. Moreover, the larger the numerical value of the primary moduli, the less the absolute value of the peak normal stresses. It should be reiterated that the peaks which do occur at the ends of the overlap (points A and B) are identical for similar adherends. For dissimilar adherends, however, the numerically largest normal stress always occurs in the adhesive at the edge of the overlap adjacent to the adherend with the lower value of flexural stiffness. The relative difference in these peaks is a function of the relative flexural stiffness of the two adherends.

The maximum adhesive normal stress occurs adjacent to the adherend of

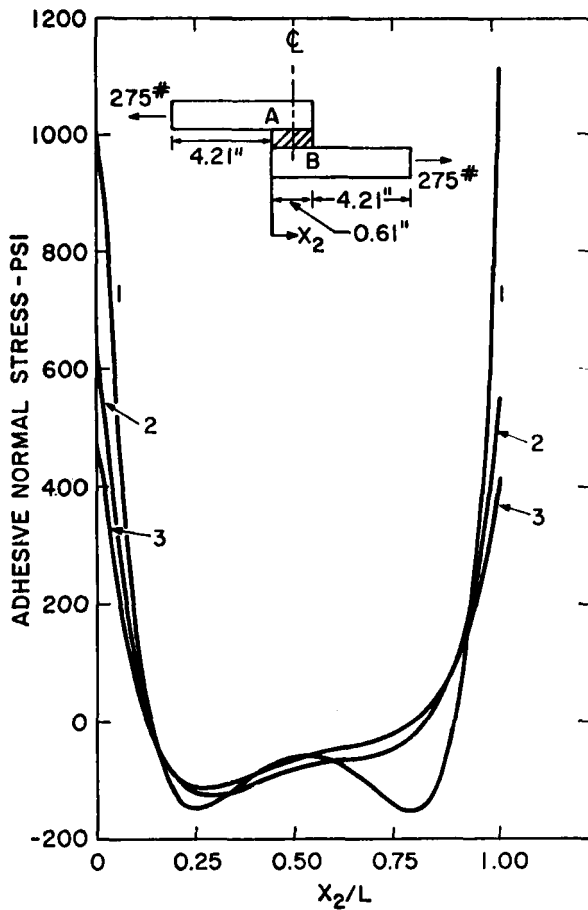


FIGURE 4 Curves 1, 2 and 3. The influence of the ratio of the primary moduli, $Q_{11(2)}/Q_{11(1)}$, of 0.148, 1.7, 4.42 respectively.

lesser flexural stiffness since that adherend will exhibit the larger deformation normal to the applied load. This gives rise to a large lateral deformation ($w_2 - w_1$) term which introduces a large normal stress in the adhesive as given in equation (15). Additionally, inspection of Eq. (15) shows that the modulus of the adhesive determines the absolute value of these normal stresses.

$$\sigma_0 = \frac{E_{\text{eff.}}(w_2 - w_1)}{\eta} \quad (15)$$

To conclude this discussion, one should note that an analogous effect to that just described can be obtained by adjusting the thickness of the adherends and/or the orientation of the plies in a laminated adherend. Both effectively alter the inplane and flexural stiffness of the adherends.

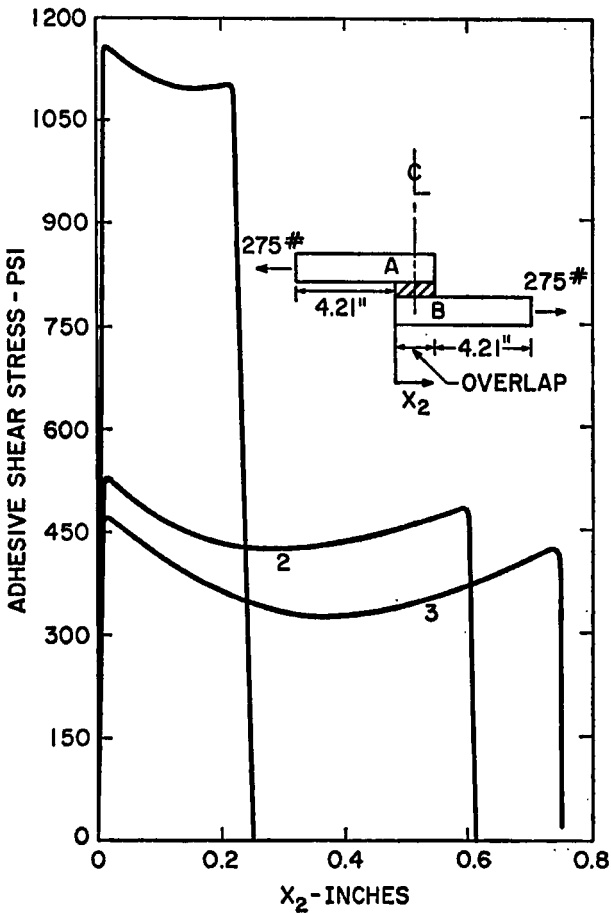


FIGURE 5 Curves 1, 2 and 3. The influence of overlap lengths of 0.25, 0.61 and 0.75 inches respectively.

The influence of altering the overlap length is often considered a convenient means to minimizing the peak adhesive stresses. The influence of overlap length upon adhesive stresses is shown in Figures (5), (6) and (7). The overall effect is obvious. As one increases the overlap length (for a constant value of L_1 and L_2 —see Figure 2), the adhesive shear stress is reduced. However, it is evident that beyond a certain overlap length, one reaches a point of diminishing returns. For example, in Figure (5), a much greater reduction in peak shear stress as well as a more uniform stress distribution occurs for an overlap change of from 0.25 inches to 0.61 inches than from 0.61 inches to 0.75 inches. Moreover, beyond a length of overlap to adherend thickness ratio of approximately 10–12, failure within the

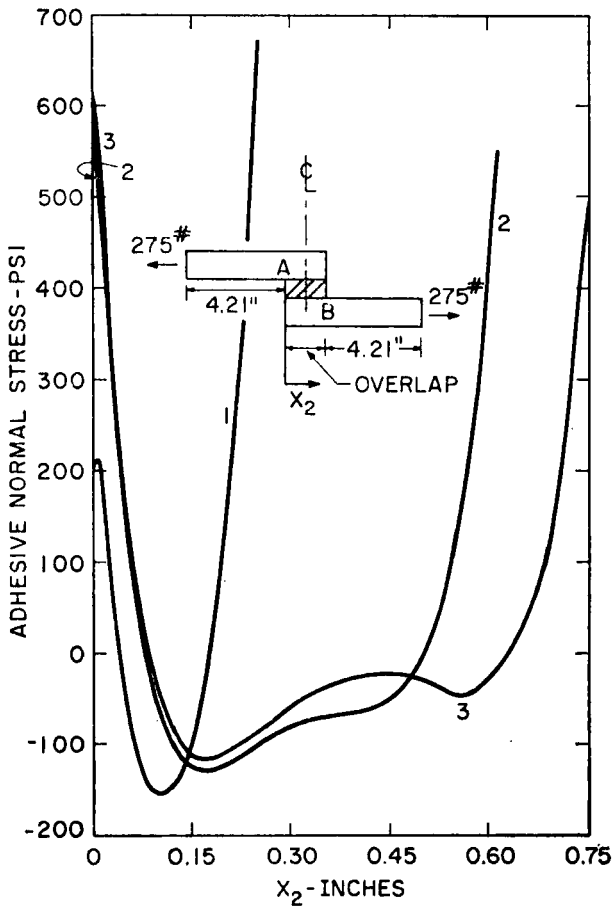


FIGURE 6 Curves 1, 2 and 3. The influence of overlap lengths of 0.25, 0.61 and 0.75 inches respectively.

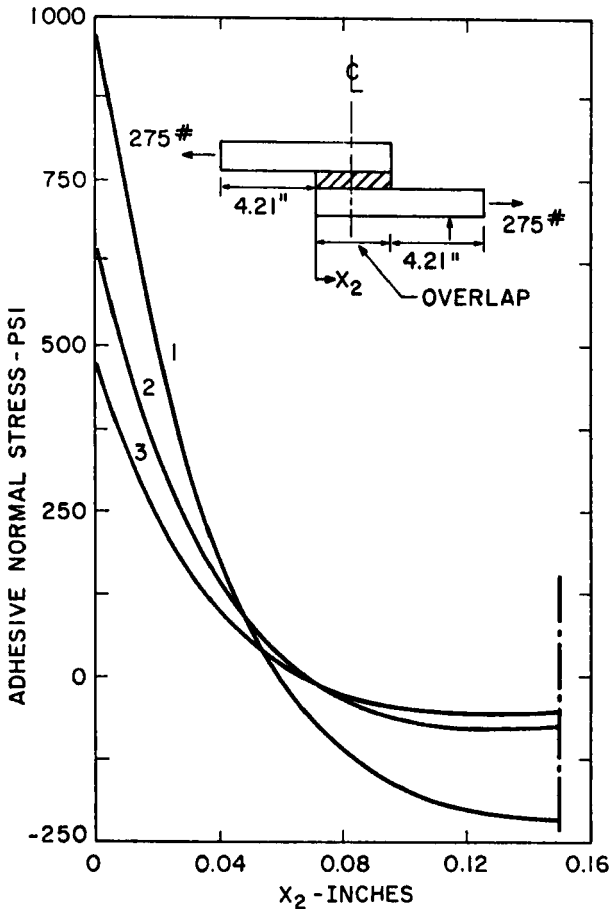


FIGURE 7 Curves 1, 2 and 3. The influence of overlap lengths of 0.31, 1.00 and 2.00 inches respectively (identical adherends).

adherend often occurs first. This can be due partially to the high edge moments induced at the edge of the overlap due to the joint eccentricity (points A and B), or for a laminated adherend, the result of high interlaminar stresses in an angle ply lamina (Ref. 2). For example, if the minimum adherend thickness is 0.063 inches, then for an overlap length greater than 0.60 inches, a failure in the adherend would likely occur first. This would negate the advantages of using a longer overlap length to reduce the peak stresses in the adhesive.

Reflecting momentarily on Figure (5), it may be observed that the maximum edge moment occurs in the adherend of maximum flexural stiffness and adjacent to the adhesive (point B), while the peak shear stress occurs in the adhesive

adjacent to the adherend of minimum in-plane stiffness. The reasons for this are identical to those given earlier to explain the impact of varying the adherends primary moduli (Q_{11}).

Figure (6) indicates little net change in the maximum adhesive normal stress for various overlap lengths for dissimilar adherends. It further indicates that while the peak normal stress is reduced on one end, it is increased at the

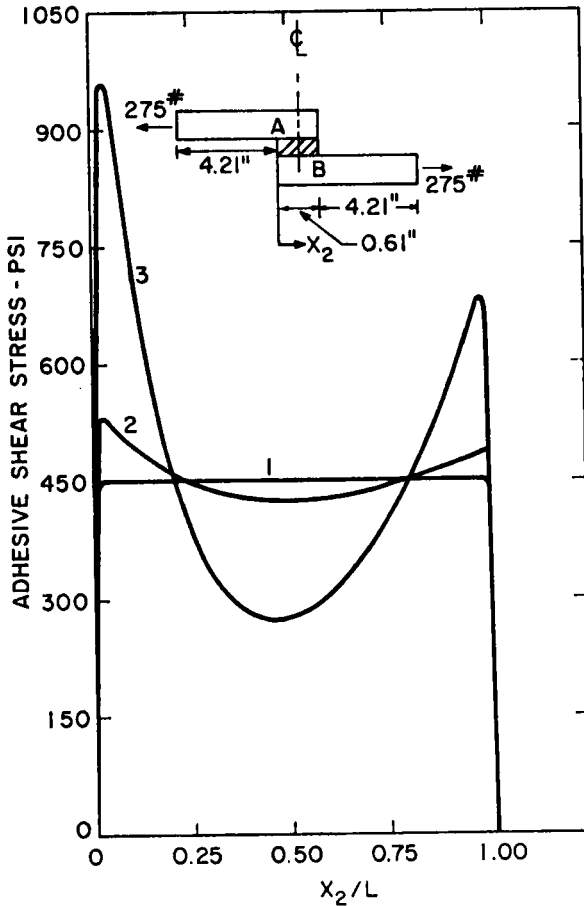


FIGURE 8 The influence of the ratio of in-plane stiffness to the effective shear factor, $(A_{11(2)}/G_{EFF.7})$ for values of $(25.6, 0.25 \text{ and } 0.025) \times 10^6$ associated with curves 1 to 3 respectively.

other. However, increasing the overlap length tends to offset the skew effect introduced by dissimilar adherends and results in the peak normal stresses being approximately equal. Moreover, if one were to design a joint with

identical adherends in which the peak normal adhesive stress is equal at each end, Figure (7) shows that a definite reduction in these stresses is possible.

Another practical way to adjust the stress distribution in an adhesive is to modify the stiffness of the adhesives shear modulus. Figure (8) shows that the larger the ratio of adherend in-plane stiffness to effective shear factor

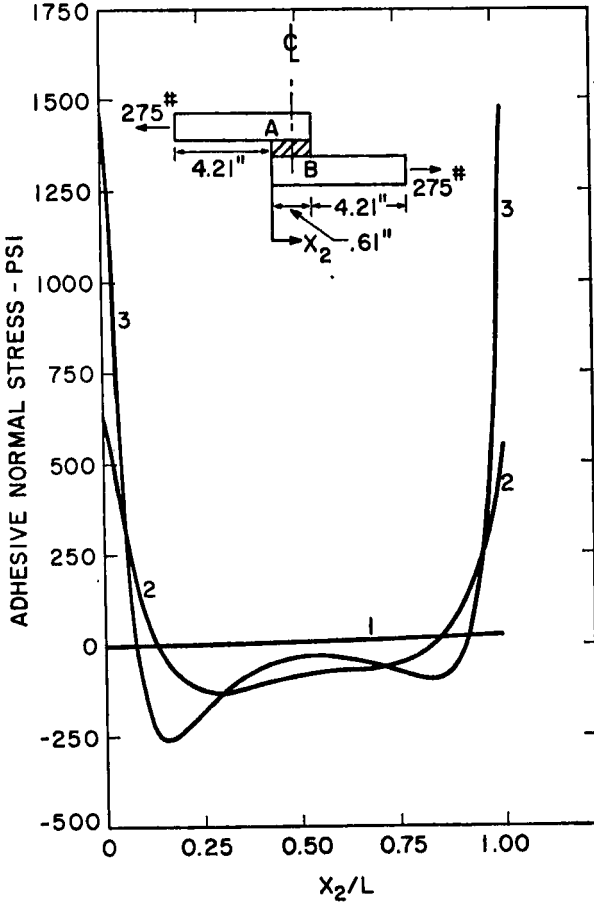


FIGURE 9 The influence of the ratio of flexural stiffness vs. the tensile factor of the adhesive, $(D_{11(2)}/E_{EFF} \cdot \eta^3)$ for values of $(14,500, 14.5 \text{ and } 1.45) \times 10^6$ associated with curves 1 to 3.

of the adhesive $(A_{11(2)}/G_{6m} \cdot \eta)$ (i.e., the softer the adhesive), the more nearly one approaches a state of pure shear in the adhesive. This is reasonable as a softer adhesive allows the load transfer into the joint to distribute in a more uniform manner. Further inspection of Figure (8) would indicate that the

maximum adhesive shear stress occurs at the edge of the overlap adjacent to the adherend with the lesser in-plane stiffness. This is identical to that observed for the primary modulus results.

For the normal range of shear moduli possessed by adhesives, its influence on the normal stress distribution is negligible. However, for extremely "soft" adhesives, the low shear modulus can result in higher normal stresses in the adhesive.

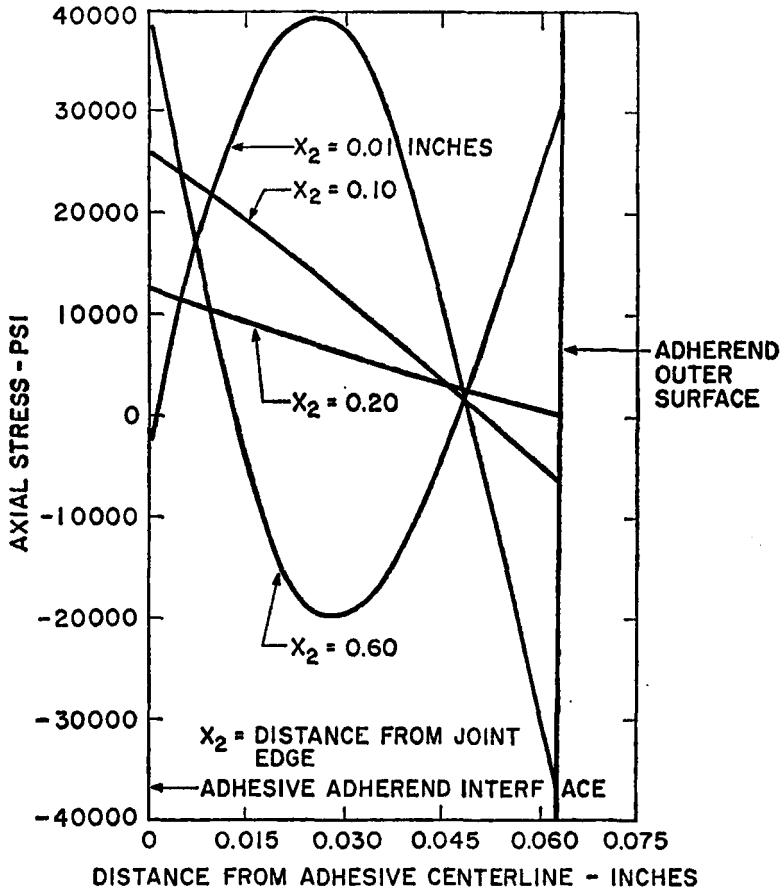


FIGURE 10 Axial stress vs. adherend thickness.

One can also select an adhesive with a specific tensile modulus. The primary impact of tensile modulus is upon the peak adhesive normal stress. This effect is seen in Figure (9) for the ratio of the flexural stiffness of adherend (2) to the effective tensile factor of the adhesive ($D_{11(2)}/E_{\text{eff}} \cdot \eta^3$). Fundamentally it shows that the larger the ratio of ($D_{11(2)}/E_{\text{eff}} \cdot \eta^3$) the smaller

the peak normal stresses. For a given adherend material such a condition is most readily met with an extremely "soft" adhesive. Further, the results again show the maximum stress to occur at the edge of the adhesive adjacent to the adherend with the minimum flexural stiffness. The larger the difference

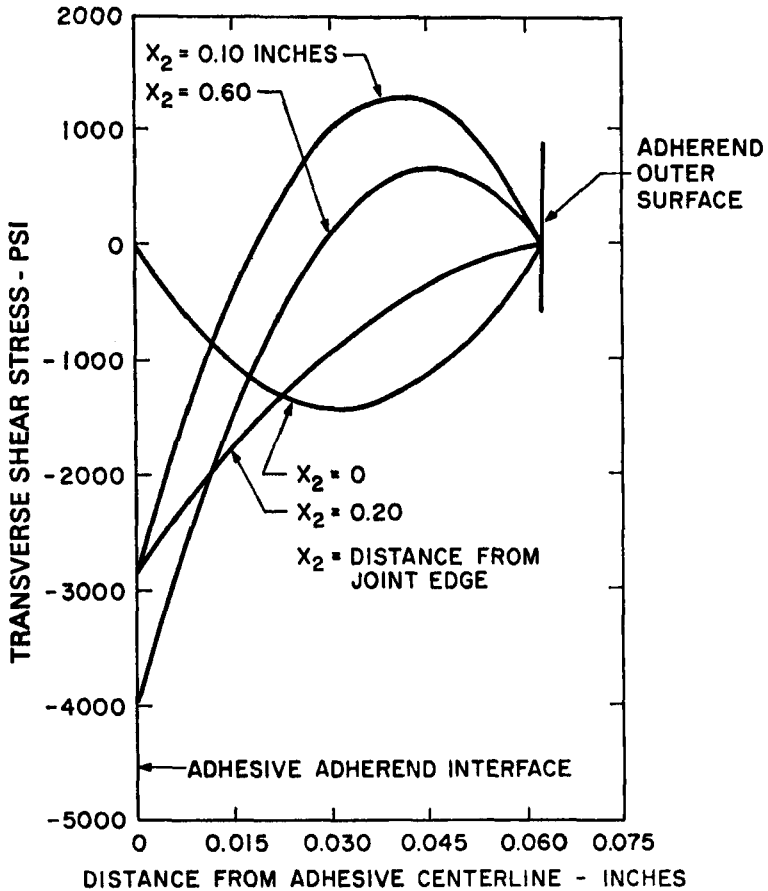


FIGURE 11 Transverse shear stress vs. adherend thickness.

in the flexural rigidity of the two adherend material systems, the more disproportionate the peak stresses at each end will be. Again, the magnitude of the peak normal stresses is a function of the "give" in the adhesive material in the direction normal to the applied load.

The influence of Q_{55} , the transverse shear modulus of the adherend, is introduced into the analysis by the transverse shear stress term τ_{xz} . Its influence on the peak shear and normal stresses is directly related to the ratio

of the primary in-plane modulus to the transverse shear modulus ($Q_{11}/2Q_{55}$) of the adherend material. Results strongly indicate a minimal influence of this parameter on the adhesive peak stresses. Therefore, its influence on the bonded joint problem would be most directly related to the stress levels the adherends experience when loaded. Such an influence could be studied because the present analytical method can calculate the stress distribution throughout the thickness of the adherends.

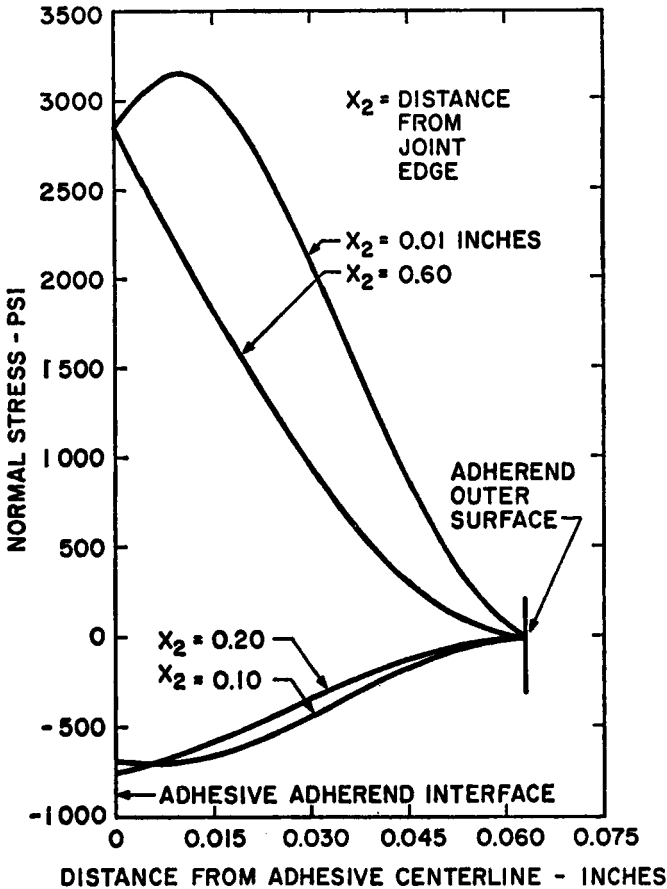


FIGURE 12 Normal stress vs. adherend thickness.

The influence of varying (L_1) and (L_4) as shown in Figure 2 and the angle of lamina orientation in laminated adherends was also investigated. No appreciable influence on the adhesive stress distribution was found for the normal range of values these variables would assume. However, lamina orientation can be most influential in the adherends failing before the adhesive.

Typical distributions of axial stress, transverse shear stress, normal stress and longitudinal stress through the adherend thickness at various locations along the overlap length are shown in Figures 10 through 13 respectively. The results shown are for two identical adherends and are meant to qualitatively show how the various stresses vary along the overlap length and throughout the adherend thickness. The overlap length is 0.31 inches.

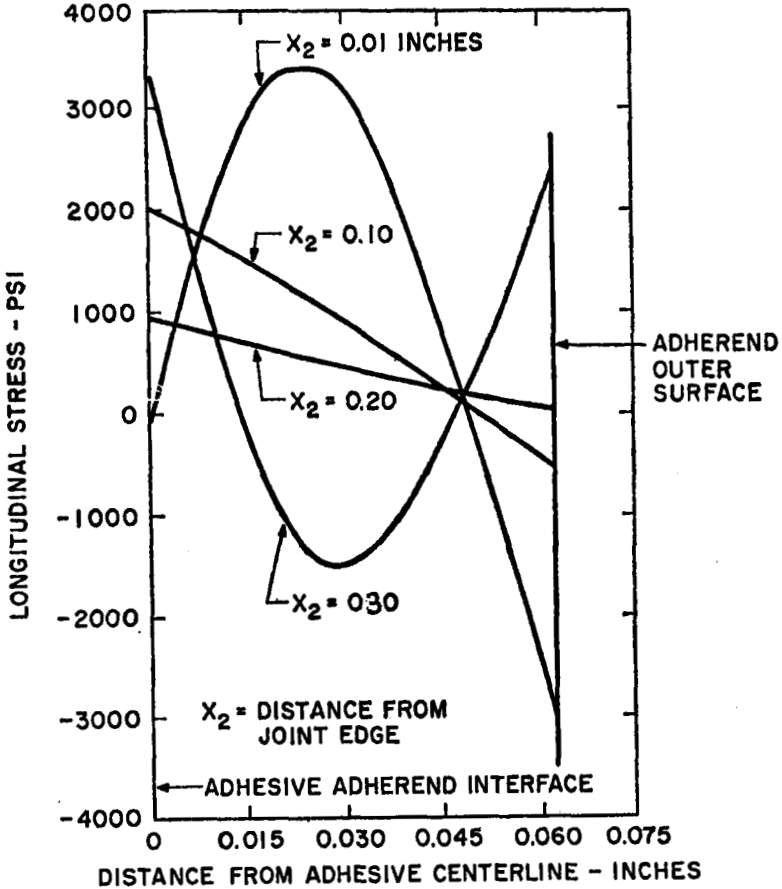


FIGURE 13 Longitudinal stress vs. adherend thickness.

Inspection of Figures 10 through 13 readily reveals that the peak stresses occur near the ends of the overlap ($x = 0.01$ and $x = 0.30$) where the moments, axial load and shears are largest. Moreover, all vary quite rapidly through the thickness of the adherend. In the central part of the overlap ($x = 0.10$ and $x = 0.20$), one sees that the peak stress levels and range of stresses are of a much less severe nature. Moreover, variation of these

stresses through the adherend thickness is less rapid than at the ends of the overlap.

While the longitudinal stresses, those in the y -direction, are relatively small, it is believed that for certain geometries of laminated adherends, the large tensile and compressive axial stresses, those in the x -direction, near the ends of the overlap in combination with the peak transverse shear and normal stresses are responsible for the angle ply of a laminated adherend adjacent to the adhesive failing before the adhesive itself (Ref. 2). Obviously, a much more thorough study of the variation of these peak stresses is desirable if one wishes to maximize the life of a joint for both static and fatigue loadings.

DESIGN RECOMMENDATIONS

Based on the results presented, certain significant design recommendations can be given for maximizing the static and fatigue load carrying capability of an adhesive bonded joint. This is best achieved by minimizing the edge shear and moment concentrations at the ends of the overlap (points A and B) and the peak adhesive shear and normal stresses. By reducing the peak adhesive stresses, the shear stress distribution in the adhesive becomes more uniform and the normal stresses, especially at the joint edges, reduce to insignificant values.

The following variables are deemed most influential and should be adjusted as specified to maximize the life of a bonded joint.

- 1). Whenever possible, join identical adherends of a like geometrical configuration. For dissimilar adherends, this can be accomplished by equalizing the in-plane and bending stiffness parameters. This minimizes the skewing of the adhesive peak shear and normal stresses seen in the previous figures and the moment and shear concentration at the edges of the joint which can lead to premature adherend failures.

- 2). Use material systems with relatively high values of primary modulus (Q_{11}). Such a system minimizes peak stress levels, yielding a more uniform adhesive shear stress distribution. It also negates to a degree the high peak stresses that occur when one either joins dissimilar adherends or uses rigid adhesives. When the adherends used have relatively low values of Q_{11} , the adhesive stress peaks can be minimized by increasing the adherend thickness.

- 3). Use an overlap length of about ten times the minimum thickness adherend. This too gives a more uniform adhesive shear stress distribution without causing the failure mode to shift into the adherend.

4) Use an adhesive with low values of effective shear and tensile moduli. Moreover, selection of the adhesive should be further influenced by the joint's intended loading history. If static, the adhesive should possess relatively high tensile and shear ultimate strength values. If the application is that of fatigue, the fracture toughness of the adhesive must be an added consideration.

5) If the adherend is laminated, the bending-stretching coupling matrix should be zero. Also, orient the plies at small angles only, to minimize the chance of interlaminar shear failures in the adherend.

6) Having designed a joint, check the stress levels in the adherends at the edge of the joint and the adhesive stress distributions to confirm that both are adequate for the static and/or fatigue loading history they will experience.

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Nomenclature

A_{11} = the in-plane stiffness of the adherend

$$= \sum_{i=1}^n (Q_{11})_i (h_{i+1} - h_i)$$

D_{11} = the flexural stiffness of the adherend

$$= \frac{1}{3} \sum_{i=1}^n (Q_{11})_i (h_{i+1}^3 - h_i^3)$$

Q_{ij} = terms of the lamina stiffness matrix referred to the material principal axes.

$[T]$ = transformation matrix

ΔT = change in temperature

Subscript T = total strain

\bar{Q}_{ij} = terms of the lamina stiffness matrix referred to an arbitrary set of axis
($[T]^{-1}[Q_{ij}][T]$)

$G_{\text{eff}} \cdot \eta$ = the adhesive effective shear factor

α_i = coefficient of thermal expansion

σ_i = normal stresses

τ_{ij} = shear stresses ($i \neq j$)

ϵ_i = normal strains

ϵ_{ij} = shear strains ($i \neq j$)

u = the adherends longitudinal displacement in the direction of the applied load

w = the adherends lateral displacement in the z -direction

1, 2, 3 axes = principal material axes

x, y, z axes = an arbitrary set of orthogonal axes conforming to the geometrical axes of the joint

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